Predictability in Complex Adaptive Systems

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Outline

- Prediction and Predictability
- Impossibility and Infeasibility
- Complexity and Complicatedness
  - Complexity
  - Asynchrony
  - Wickedness

https://www.jasss.org/24/3/2.html
Take-home messages

1. Prediction in complex systems is theoretically possible, but pragmatically infeasible
   - Typically
   - Under a specific definition of ‘complex’

2. Asynchrony adds exponentially to the infeasibility

3. Wickedness renders prediction largely irrelevant where it entails terminological transformation
Prediction

- Prediction
  - “the ability to reliably anticipate well-defined aspects of data that is not currently known to a useful degree of accuracy via computations using the model”
    - Edmonds et al. (2019, para. 2.4)

- Useful
  - Subjective criterion
  - Modality of usefulness:
    - Necessarily useful – all stakeholders care about predicted model state
    - Possibly useful – some stakeholders care
    - Necessarily not useful – no stakeholders care
  - Assume:
    - Each stakeholder cares about at least one model state
Predictability

- Matching models:
  - A population of models that fit the empirical data
- Four kinds of predictability:
  - **Invariably predictable**: All matching models predict the same state
  - **Omissively predictable**: At least one state is not predicted by any matching model
  - **Asymmetrically unpredictable**: All states are possible, but different numbers of match models predict them
  - **Symmetrically unpredictable**: All states are possible; each having the same number of matching models
- Two scales
  - Individual: State spaces of individuals (e.g. agents)
  - System: State spaces of the whole system (population + environment)
Predictability

- **Matching models:**
  - A population of models that fit the empirical data

- **Four kinds of predictability:**
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Impossibility and Infeasibility

- Turing Machine (TM)
  - Basic theoretical construct in computer science
  - Finite set of internal states
  - Unbounded tape of cells
    - Finite alphabet of symbols, one per cell
  - Transition table
    - Internal state + symbol read from tape → new state + symbol to write + move tape L or R

- A TM can be built to do anything computable
  - Church-Turing thesis

- Impossible
  - Can’t design a TM to do it

- Infeasible
  - Can design a TM, but:
    - Might require more atoms to build it than there are in the universe (too much memory)
    - Might require more time to run the program than the life of the universe (too much time)
Complexity and Complicatedness

- **Complex**
  - Santa Fe Institute conceptualization
    - “Large networks of components with no central control and simple rules of operation”
      - Mitchell (2009, p. 13)
    - Andersson et al. (2014) point out that those components are of the same type

- **Complicated**
  - Lots of different kinds of component
  - Decomposable functionality
  - ‘Wicked’ (Andersson et al. 2014)
    - Complex and complicated
    - High uncertainty

Societal systems – Complex or worse?
Claes Andersson 1, 2, Anton Törnberg 3, Petter Törnberg 3

Highlights
- Societal systems are both complex and complicated.
- They are recalcitrant to complexity science like any formal approach.
- “Wicked systems” outlined – complicated and complex, different from both.
- Combining approaches to complexity and complicatedness is thereby highly challenging.
Mathematical Issues in Complex Systems

- Nonlinearity (p. 8)
- Non-ergodicity (p. 350)
  - Path dependence means law of large numbers does not hold (p. 85)
- Non-Markovian (p. 350)
- Out-of-equilibrium (p. 317)
Complex systems and predictability

Invariably Predictable

Omissively Predictable

Asymmetrically Unpredictable

Symmetrically Unpredictable

[exclude any state]

[only one possible state]

[else]

[all states appear with roughly equal probability]

[all system states equally possible]

[all individuals Invariably Predictable]

[only one possible state]

[else]

[model search space smaller than system state space]

[else]

[all system states equally possible]

[else]

[all states appear with roughly equal probability]

[all system states equally possible]

[exclude any state]

[only one possible state]
Complex systems and predictability

Invariably Predictable  Omissively Predictable  Asymmetrically Unpredictable  Symmetrically Unpredictable

Individual

System

Ergodic and therefore not complex?
Are complex systems really predictable?

- Example of cellular automata (CA)
  - Defined by
    - Cell state alphabet
    - Neighbourhood
    - Cell transition rule
  - Certain transition rules classed as ‘complex’
    - Wolfram (1984): Class 4 (complex) CAs unpredictable except by simulation
    - Cook (2004): Rule 110 elementary CA capable of universal computation

- CA data generator
- Search transition rule space
- Run rules that match the data

[Diagram of 'Elementary' Cellular Automaton (Four matching transition rules)]

% of runs with n-eliminated transition rules

[max-data] 

[Data visualization]
But...

- Predictability based on transition rule space search
  - Polhill et al. (2021) paper shows that this space is finite for CAs, and even asynchronous TMs
    - Assumes knowledge of alphabet and for TMs, the number of internal states
  - So, not impossible...

- Transition rule space can be very big
  - Exhaustive search infeasible
    - E.g. $\sim 10^{154}$ possibilities for family of 2D CAs to which Conway’s Game of Life belongs
  - ...but infeasible
Asynchrony

- System state is vector of states of components
  - Complex (and complicated) systems have lots of components
- Scheduling determines how system state at time $T$ is computed from state at $T - 1$
  - Which component calculates its behaviour when, and using which data?
- What if scheduling is non-computable?
  - Even if all matching models agree about state at time $T$, to calculate state at time $T + n$:
    - Have to explore all possible orderings of components behaving
    - Exponential in $n$ and number of components
    - Partial computability (if unknown) means we might explore options for ordering that would not happen
- Asynchrony adds exponentially to the problem of prediction
Wickedness

- Andersson & Törnberg (2018) refer to ontological uncertainty in wicked systems
  - “emergence of qualitative novelty”
- New vocabulary in transformed systems
  - No existing data uses that vocabulary
  - For each model that fits the data $K$ and $A$:
    - Can include (tentative) new vocabulary $K^+$ and $A^+$
    - But every possible transition rule involving $K^+$ and $A^+$ will fit!
    - All future states then equally likely
      - Symmetric unpredictability
      - Necessarily NOT useful
- In wicked systems, prediction is useless as the system evolves away from the data’s vocabulary
  - N.B. Some definitions of complexity would include this kind of evolution as a feature
Conclusion

- When we think about different kinds of predictability, complex systems should be at least omissively predictable because they are non-ergodic.

- However, exhaustively searching the space of possible transition functions is infeasible for all but the simplest cellular automaton.

- Asynchrony makes computing the prediction itself infeasible due to combinatorial explosion in options needing to be explored.

- Evolution of systems away from vocabulary used for observed data makes prediction useless because all future states are equally likely from matching models.
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References
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